## Huygens Principle, reflection, refraction and diffraction

In the mass-spring analog of wave motion it might be argued that a compressed spring at some point in the chain was the initiator of the forces and displacements that continued in the direction of propagation of the wave. After all the adjacent mass can't really 'tell' whether the force applied to it was coming from a progressing wave or from an 'external' source. Each of these virtual sources gives a push which keeps the wave going in the same direction. Huygens reasoned that the same was true in a solid. He stated that each point on a wave front could be considered as the source of a new spreading wave in the existing wave direction. It is as if the incident wave excites a particular element or particle which then becomes the source of a new spreading wave. Subsequent configurations of the wave front can then be obtained by summing up the contributions from all these secondary sources (also called point scatterers). Huygens Principle is a perfectly sound physical description of the propagation process as well as being a very useful mathematical representation.

The application of Huygens Principle to a spherically spreading wave or to a plane wave in a homogeneous medium, is somewhat trivial. The following cartoon shows several of the continuously distributed 'sources' on a wave front at some time t and the effect of summing all of their wave fronts a short time later to produce the new wave front. A detailed analysis of the summation process would show that all the lateral contributions from each new point source cancel in the summation leaving only the radial particle motions of the initial wave.
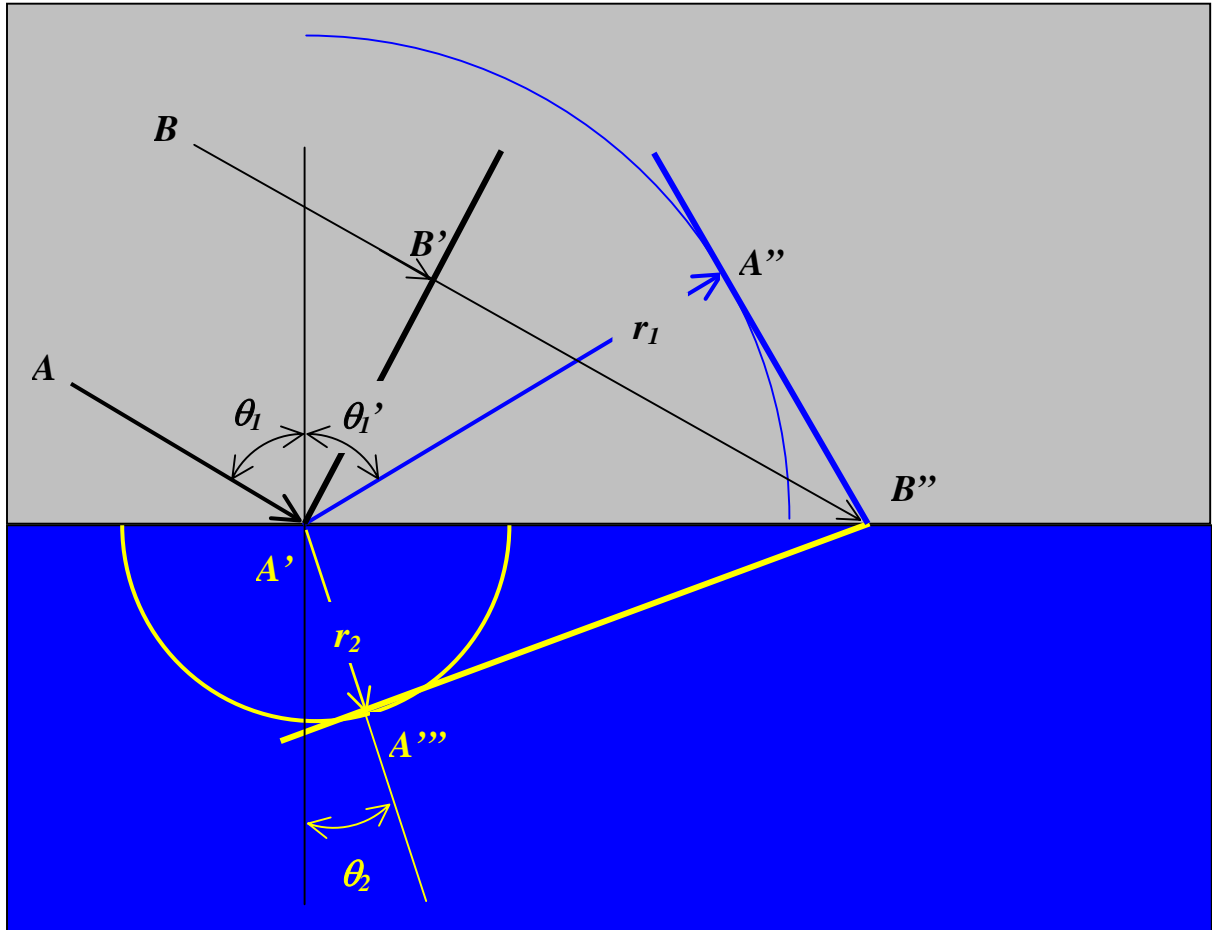
Wave front at t

Wave front at t +Δt
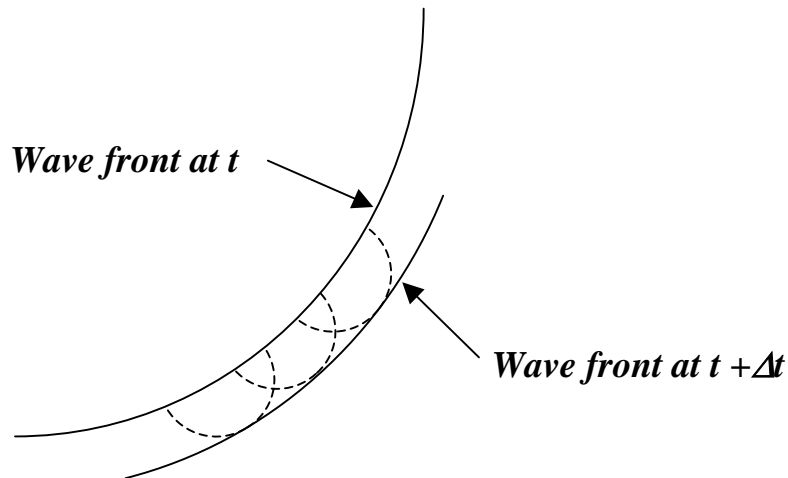
Note again that this process keeps pushing the wave in the direction of propagation. Only the portions of the wave fronts in the positive radial direction sum constructively.

Snell's Law

The most useful application of Huygens' Principle is for describing the wave fronts created when a wave is incident on a planar interface between two media of different velocity. In the following cartoon a plane wave with wave front A'B' is incident on the horizontal interface between two media of velocity V_1 and V_2. AA' and BB' are two representative rays in the incident wave. The contrast in physical properties actually causes the elementary Huygens sources to send waves in both directions from the interface. The *angle of incidence*, θ_1, is defined as the angle between the ray and the normal to the interface. Following Huygens' Principle we consider that point A' is the source of a new wave which begins propagating in the first medium with velocity V_1 and in the second medium with velocity V_2. At subsequent times the incident field starts similar waves at successive

points along the interface. At a time, $t_{B'B''}$, given by $B'B''/V_1$, point B' on the incident wave front has reached the interface. During this time the secondary Huygens wave fronts sum to yield the new wave fronts $B''A''$ and $B''A'$.

From the above geometry it can be seen that:

Sin θ_1=B'B"/A'B" = $V_1\ t_{B'B''}$ /A'B"

and

Sin θ_1'= r_1/A'B" = $V_1\ t_{B'B''}$ /A'B"

So we find the almost obvious result that $\theta_1 = \theta_1'$.

Note here that if the incident disturbance had given rise to *another wave type* of velocity V_{1S} at A', then another wave front would be generated a ray of which would have progressed a distance $V_{1S}\, t_{B'B''}$ back into medium 1 during the time the incident disturbance reached B''. Denoting its angle of reflection as θ_{1S} the above relationships show that:

$$\text{Sin } \theta_{1S} / V_{1S} = \text{Sin } \theta_1 / V_1$$

The wave front in the lower medium has progressed to radius r_2 in time $t_{B'B''}$, and so

$$\text{Sin } \theta_2 = r_2 / A'B'' = V_2\, t_{B'B''} / A'B''$$

From which we can deduce the general relation that

$$\text{Sin } \theta_1 / V_1 = \text{Sin } \theta_2 / V_2$$

For *any* reflected or refracted waves. This is Snell's Law.